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NONLOCAL BOUNDARY VALUE PROBLEM FOR A NONLINEAR IMPULSIVE INTEGRO-DIFFERENTIAL SYSTEM WITH MAXIMA

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Abstract: *A nonlocal boundary value problem for a first order system of ordinary integro-differential equations with impulsive effects and maxima is investigated. The boundary value problem is given by the integral condition. The method of successive approximations in combination it with the method of compressing mapping is used. The existence and uniqueness of the solution of the boundary value problem are proved. The continuous dependence of the solutions on the right-hand side of the boundary condition is showed.*

Аннотация: *Исследуется нелокальная краевая задача для системы обыкновенных интегро-дифференциальных уравнений первого порядка с импульсными эффектами и максимумами. Краевая задача задается интегральным условием. Используется метод последовательных приближений* *в сочетании с методом сжимающего отображения. Доказаны существование и единственность решения краевой задачи. Показана непрерывная зависимость решений от правой части граничного условия.*

Annotatsiya: *Impuls ta'sirli va maksimallari bo'lgan birinchi tartibli oddiy integral-differensial tenglamalar sistemasi uchun nolokal chegaraviy masalani o'rganamiz. Chegaraviy masala integral shart bilan beriladi. Ketma-ket yaqinlashish usuli qisqartirib akslantirish usuli bilan birgalikda qo'llaniladi. Chegaraviy masala yechimining mavjudligi va yagonaligi isbotlangan. Yechimlarning chegaraviy shartning o'ng tomonida uzluksiz bog'liqligi ko'rsatilgan.*

Key words: *impulsive integro-differential equations, nonlocal boundary condition, successive approximations, existence and uniqueness of solution, continuous dependence of solution.*

Ключевые слова: *импульсные интегро-дифференциальные уравнения, нелокальные граничные условия, последовательные приближения, существование и единственность решения, непрерывная зависимость решения.*

Kalit so'zlar: *impulsli integral-differensial tenglamalar, nolokal chegaraviy shartlar, ketma-ket yaqinlashishlar, yechimning mavjudligi va yagonaligi, yechimning uzluksiz bog'liqligi.*

Introduction

Many problems in modern sciences, technology and economics are described by differential equations, the solution of which is functions with first kind discontinuities at fixed or non-fixed times. Such differential equations are called differential equations with impulse effects [1-3]. In recent years the interest in the studing of differential equations with nonlocal boundary value conditions is increasing (see, for example, [4-5]). Also a lot of publications of studying on differential equations with impulsive effects, describing many natural and practical processes, are appearing [6-7].

In this paper, we investigate a nonlocal boundary value problem for a system of ordinary first order differential equations with impulsive effects and maxima. The questions of the existence and uniqueness of the solution to the boundary value problem, as well as the continuous dependence of the solution on the right-hand side of the boundary condition, are investigated. We note, that the differential equations with maxima play an important role also in solving control problems of the sale of goods and investment of manufacturing companies in a market economy [8]. In it is justified that the theoretical study of differential equations with maxima is relevant.

Problem statement

On the segment $[0,T]$ we consider the following first order system of nonlinear integro-differential equations

$$x'(t) = f\left(t, x(t), \int_0^T \Theta\left(t, s, \max\left\{x(\tau) \mid \tau \in \left[\lambda_1 s; \lambda_2 s\right]\right\}\right) ds\right), \; t \neq t_i, \; i = 1, 2, ..., p \quad (1)$$

with nonlocal boundary value condition

$$Ax(0) + \int_0^T K(t,s)\, x(s)\, ds = B(t) \quad (2)$$

and nonlinear impulsive effect

$$x\left(t_i^+\right) - x\left(t_i^-\right) = I_i\left(x\left(t_i\right)\right), \quad i = 1, 2, ..., p, \quad (3)$$

where $0 = t_0 < t_1 < ... < t_p < t_{p+1} = T$, $A \in R^{n\times n}$ is given matrix, $K(t,s)$ is given $n\times n-$dimensional matrix function and $\det Q(t) \neq 0,\ Q(t) = A + \int_0^T K(t,s)ds,$ $f:[0,T]\times R^n \times R^n \to R^n$, $\Theta:[0,T]^2 \times R^n \to R^n$, $I_i : R^n \to R^n$ are given functions; $0<\lambda_1<\lambda_2<1$, $x(t_i^+) = \lim_{h\to 0^+}(x_i + h)$, $x(t_i^-) = \lim_{h\to 0^-}(t_i - h)$ are right-hand sided and left-hand sided limits of function $x(t)$ at the point $t = t_i$, respectively.

By $C([0,T], R^n)$ denoted the Banach space, which consists of continuous vector functions $x(t)$, defined on the segment $[0,T]$, with values in R^n and with the norm

$$\| x \| = \sqrt{\sum_{j=1}^{n} \max_{0\le t\le T} | x_j(t) |}.$$

By $PC([0,T], R^n)$ denoted the linear vector space

$$PC([0,T], R^n) = \left\{ x:[0,T]\to R^n;\ x(t)\in C\left((t_i, t_{i+1}], R^n\right),\ i=1,...,p \right\},$$

where $x(t_i^+)$ and $x(t_i^-)$ $(i = 0,1,...,p)$ exist and bounded; $x(t_i^-) = x(t_i)$.

Note, that the linear vector space $PC([0,T], R^n)$ is Banach space with the following norm

$$\| x \|_{PC} = \max\left\{ \| x \|_{C((t_i, t_{i+1}])},\ i = 1,2,...,p \right\}.$$

Formulation of problem. To find the function $x(t) \in PC([0,T], R^n)$, which for all $t\in[0,T],\ t\neq t_i,\ i = 1,2,...,p$ satisfies the integro-differential equation (1), nonlocal integral condition (2) and for $t = t_i$ $i = 1,2,...,p,$ $0 < t_1 < t_2 < ... < t_p < T$ satisfies the nonlinear limit condition (3).

Reduction to an integral equation

Let the function $x(t) \in PC([0,T], R^n)$ is a solution of the nonlocal boundary value problem (1)-(3). Then by integration of the equation (1) on the interval $t \in (0, t_{i+1}]$, we obtain

$$\int_0^t f(s,x,y)\,ds = \int_0^t x'(s)\,ds = \left[x(t_1) - x(0^+) \right] + \left[x(t_2) - x(t_1^+) \right] + ... + \left[x(t) - x(t_i^+) \right] =$$

$$= -x(0) - \left[x(t_1^+) - x(t_1) \right] - \left[x(t_2^+) - x(t_2) \right] - ... - \left[x(t_i^+) - x(t_i) \right] + x(t).$$

Taking into account the condition (3), the last equality we rewrite as

$$x(t) = x(0) + \int_0^t f(s,x,y)\,ds + \sum_{0<t_i<t} I_i\left(x(t_i)\right). \quad (4)$$

We subordinate the function $x(t) \in PC([0,T], R^n)$ in (4) to satisfy the boundary value condition (2):

$$\left[A+\int_0^T K(t,s)ds\right]x(0)=$$

$$=B(t)-\int_0^T K(t,s)\int_0^s f(\theta,x,y)d\theta ds-\int_0^T K(t,s)\sum_{0<t_i<t} I_i\left(x\left(t_i\right)\right)ds. \quad (5)$$

By virtue of $\det Q(t)=\det\left[A+\int_0^T K(t,s)ds\right]\neq 0$, the equality (5) we rewrite as

$$x(0)=Q^{-1}(t)\left[B(t)-\int_0^T K(t,s)\int_0^s f(\theta,x,y)\,d\theta ds-\int_0^T K(t,s)\sum_{0<t_i<t} I_i\left(x\left(t_i\right)\right)ds\right]. \quad (6)$$

Substituting equality (6) into representation (4), we obtain

$$x(t)=Q^{-1}(t)\left[B(t)-\int_0^T K(t,s)\int_0^s f(\theta,x,y)\,d\theta ds-\int_0^T K(t,s)\sum_{0<t_i<t} I_i\left(x\left(t_i\right)\right)ds\right]+$$

$$+\int_0^t f(s,x,y)ds+\sum_{0<t_i<t} I_i\left(x\left(t_i\right)\right). \quad (7)$$

Since the following equalities hold

$$\int_0^T K(t,s)\int_0^s f(\theta,x,y)\,d\theta ds=\int_0^T\int_s^T K(t,\theta)\,d\theta\, f(s,x,y)\,ds,$$

$$\int_0^T K(t,s)\sum_{0<t_i<t} I_i\left(x\left(t_i\right)\right)ds=\sum_{0<t_i<T}\int_{t_i}^T K(t,s)\,ds\, I_i\left(x\left(t_i\right)\right),$$

from presentation (7) we obtain

$$x(t)=Q^{-1}(t)B(t)-Q^{-1}(t)\int_0^T\int_s^T K(t,\theta)\,d\theta\, f(s,x,y)\,ds-$$

$$-Q^{-1}(t)\sum_{0<t_i<t}\int_{t_i}^T K(t,s)\,ds\, I_i\left(x\left(t_i\right)\right)+\int_0^t f(s,x,y)\,ds+\sum_{0<t_i<t} I_i\left(x\left(t_i\right)\right). \quad (8)$$

After some simplifications in representation (8) we obtain that the following equalities hold:

$$\int_0^t f(s,x,y)\,ds-Q^{-1}(t)\int_0^T\int_s^T K(t,\theta)\,d\theta\, f(s,x,y)\,ds=$$

$$=Q^{-1}(t)\int_0^t\left(A+\int_0^s K(t,\theta)d\theta\right)f(s,x,y)\,ds-$$

$$-Q^{-1}(t)\int_0^T\int_s^T K(t,\theta)\,d\theta\, f(s,x,y)\,ds; \quad (9)$$

$$\sum_{0<t_i<t} I_i\left(x\left(t_i\right)\right)-Q^{-1}(t)\sum_{0<t_i<T}\int_{t_i}^T K(t,s)\,ds\, I_i\left(x\left(t_i\right)\right)=$$

$$= Q^{-1}(t)\sum_{0<t_i<t}\left(A+\int_0^{t_i}K(t_i,s)ds\right)I_i\left(x\left(t_i\right)\right)-\sum_{t<t_{i+1}<T}Q^{-1}(t_i)\int_{t_i}^{T}K(t_i,s)\,ds\,I_i\left(x\left(t_i\right)\right). \quad (10)$$

Taking into account (9) and (10), from the presentation (8) we obtain the nonlinear equation

$$x(t)=J(t;x)\equiv Q^{-1}(t)B(t)+\sum_{0<t_i<t}G\left(t,t_i\right)I_i\left(x\left(t_i\right)\right)+$$

$$+\int_0^T G(t,s)\,f\left(s,x(s),\int_0^T\Theta\left(s,\theta,\max\left\{x(\tau)\,|\,\tau\in\left[\lambda_1\theta;\lambda_2\theta\right]\right\}\right)d\theta\right)ds \quad (11)$$

for $t\in\left(t_i,t_{i+1}\right],\ i=0,1,...,p,$ where

$$G(t,s)=\begin{cases} Q^{-1}(t)\left(A+\int_0^s K(t,\theta)d\theta\right), & 0\le s\le t,\\ -Q^{-1}(t)\int_s^T K(t,\theta)d\theta, & t<s\le T.\end{cases}$$

It is easy to verify that the equation (11) satisfies problem (1)-(3).

The questions of one value solvability

Theorem. Suppose the following conditions are fulfilled:

1). $M_f=\max_{0\le t\le T}\left|f\left(t,Q^{-1}(t)B(t),\int_0^T\Theta\left(t,s,Q^{-1}(s)B(s)\right)ds\right)\right|<\infty;$

2). $m_I=\max_{0\le t\le T}\max_{i\in\{1,2,...,p\}}\left|I_i\left(Q^{-1}(t)B(t)\right)\right|<\infty;$

3). For all $t\in[0,T],\ x,y\in R^n$ holds

$$\left|f(t,x_1,y_1)-f(t,x_2,y_2)\right|\le M_1(t)\left|x_1-x_2\right|+M_2(t)\left|y_1-y_2\right|;$$

4). For all $t,s\in[0,T]^2,\ x\in R^n$ holds

$$\left|\Theta(t,s,x_1)-\Theta(t,s,x_2)\right|\le M_3(t,s)\left|x_1-x_2\right|;$$

5). For all $x\in R^n,\ i=0,1,...,p$ holds

$$\left|I_i(x_1)-I_i(x_2)\right|\le m_i\left|x_1-x_2\right|;$$

6). $\rho=S_1+S_2+S_3<1,$ where

$$S_1=\max_{0\le t\le T}\int_0^T\left|G(t,s)\right|M_1(s)\,ds,\quad S_2=\max_{0\le t\le T}\int_0^T\left|G(t,s)\right|M_1(s)\int_0^T M_3(s,\theta)\,d\theta\,ds,$$

$$S_3=\max_{0\le t\le T}\sum_{i=1}^{p}\left|G(t,t_i)\right|\cdot m_i.$$

Then the nonlocal boundary value problem (1)-(3) has a unique solution $x(t)\in PC\left([0,T],R^n\right)$. This solution can be found by the following iterative process:

$$\begin{cases} x^k\left(t\right)=J(t;x^{k-1}),\ k=1,2,3,...\\ x^0\left(t\right)=Q^{-1}(t)B(t),\ t\in(t_i,t_{i+1}),\ i=0,1,2,...,p.\end{cases} \quad (12)$$

In addition, for this solution it is true the following estimate

$$\|x_1(t)-x_2(t)\|_{PC} \le \frac{\|Q^{-1}(t)\|}{1-\rho}\cdot\|B_1(t)-B_2(t)\|.$$

Proof. We consider the following operator

$J: PC\left([0,T];R^n\right)\to PC\left([0,T]\times R^n\right)$, defined by the right-hand side of equation (11). Appling the principle of contracting operators to (11), we show that the operator J, defined by equation (11), has a unique fixed point.

For the zero approximation of the iteration process (12) we easily obtain that

$$\| x^0(t) \| \le \| Q^{-1}(t)B(t) \| \le \| Q^{-1}(t) \| \cdot \| B(t) \| < \infty. \quad (13)$$

Taking first and second conditions of the theorem and estimate (13), for the first difference of the approximations (12) we have the following estimate

$$\| x^1(t)-x^0(t) \| \le \max_{0\le t\le T}\sum_{i=1}^{p}|G(t,t_i)| \left| I_i\left(x^0(t_i)\right)\right| +$$

$$+\max_{0\le t\le T}\int_0^T |G(t,s)| \left| f\left(s,x^0(s),\int_0^T \Theta\left(s,\theta,\max\left\{x^0(\tau)\,|\,\tau\in\left[\lambda_1\theta;\lambda_2\theta\right]\right\}\right)d\theta\right)\right| ds \le$$

$$\le S_0\left(M_f+m_I\right)<\infty, \quad (14)$$

where

$$S_0=\max_{0\le t\le T}\int_0^T |G(t,s)|\,ds+\max_{0\le t\le T}\sum_{i=1}^{p}|G(t,t_i)|,$$

$$M_f=\max_{0\le t\le T}\left| f\left(t,Q^{-1}(t)B(t),\int_0^T\Theta\left(t,s,Q^{-1}(s)B(s)\right)ds\right)\right|,$$

$$m_I=\max_{0\le t\le T}\max_{i\in\{1,2,\dots,p\}}\left| I_i\left(Q^{-1}(t)B(t)\right)\right|.$$

Then, by the third, fourth and fifth conditions of the theorem, for difference of arbitrary consecutive approximations and arbitrary $t\in\left(t_i,t_{i+1}\right]$ we have

$$\left| x^k(t)-x^{k-1}(t)\right| \le$$

$$\le\int_0^T |G(t,s)| \left| f\left(s,x^{k-1}(s),\int_0^T\Theta\left(s,\theta,\max\left\{x^{k-1}(\tau)\,|\,\tau\in\left[\lambda_1\theta;\lambda_2\theta\right]\right\}\right)d\theta\right)-\right.$$

$$\left.-f\left(s,x^{k-2}(s),\int_0^T\Theta\left(s,\theta,\max\left\{x^{k-2}(\tau)\,|\,\tau\in\left[\lambda_1\theta;\lambda_2\theta\right]\right\}\right)d\theta\right)\right| ds+$$

$$+\sum_{i=1}^{p}|G(t,t_i)| \left| I_i\left(x^{k-1}(t_i)\right)-I_i\left(x^{k-2}(t_i)\right)\right| \le$$

$$\le\int_0^T |G(t,s)| \left[M_1(s)\cdot\left| x^{k-1}(s)-x^{k-2}(s)\right| + M_2(s)\int_0^T M_3(s,\theta)\times\right.$$

$$\left.\times\left|\max\left\{x^{k-1}(\tau)\,|\,\tau\in\left[\lambda_1\theta;\lambda_2\theta\right]\right\}-\max\left\{x^{k-2}(\tau)\,|\,\tau\in\left[\lambda_1\theta;\lambda_2\theta\right]\right\}\right| d\theta\right] ds+$$

$$+\sum_{i=1}^{p}\left|G(t,t_i)\right|\cdot m_i\cdot\left|x^{k-1}(t_i)-x^{k-2}(t_i)\right|.$$

Hence, by the introduced norm we obtain

$$\left\| x^k(t)-x^{k-1}(t)\right\|_{PC}\le S_1\left\| x^{k-1}(t)-x^{k-2}(t)\right\|_{PC}+$$

$$+S_2\left\|\max\left\{x^{k-1}(\tau)\,|\,\tau\in\left[\lambda_1 t;\lambda_2 t\right]\right\}-\max\left\{x^{k-2}(\tau)\,|\,\tau\in\left[\lambda_1 t;\lambda_2 t\right]\right\}\right\|_{PC}+$$

$$+S_3\left\| x^{k-1}(t)-x^{k-2}(t)\right\|_{PC}\le\rho\cdot\left\| x^{k-1}(t)-x^{k-2}(t)\right\|_{PC},\quad(15)$$

where $\rho=S_1+S_2+S_3$,

$$S_1=\max_{0\le t\le T}\int_0^T\left|G(t,s)\right|M_1(s)\,ds,\;\; S_2=\max_{0\le t\le T}\int_0^T\left|G(t,s)\right|M_1(s)\int_0^T M_3(s,\theta)\,d\theta\,ds,$$

$$S_3=\max_{0\le t\le T}\sum_{i=1}^{p}\left|G(t,t_i)\right|\cdot m_i.$$

According to the last condition of the theorem $\rho<1$. Therefore, from the estimate (15) we have

$$\left\| x^{k-1}(t)-x^{k-2}(t)\right\|_{PC}<\left\| x^{k-1}(t)-x^{k-2}(t)\right\|_{PC}.\quad(16)$$

It follows from (16) that the operator J on the right-hand side of the equation (11) is contracting. According to fixed point principle, taking into account estimates (13)-(16), we conclude that the operator J has a unique fixed point. Consequently, the nonlocal boundary value problem (1)-(3) has a unique solution $x(t)\in PC\left([0,T],R^n\right)$.

Now let us show the continuous dependence of the solution to the boundary value problem (1)-(3) on the right-hand side of condition (2). Let $B_1(t),B_2(t)\in R^n$ are two different vector functions and $x_1(t),x_2(t)\in PC\left([0,T],R^n\right)$ are corresponding solutions of the problem (1)-(3). Then from the equation (11) we have

$$x_1(t)-x_2(t)=Q^{-1}(t)\left[B_1(t)-B_2(t)\right]+$$

$$+\int_0^T G(t,s)\left[f\left(s,x_1(s),\int_0^T\Theta\left(s,\theta,\max\left\{x_1(\tau)\,|\,\tau\in\left[\lambda_1\theta;\lambda_2\theta\right]\right\}\right)d\theta\right)-\right.$$

$$\left.-f\left(s,x_2(s),\int_0^T\Theta\left(s,\theta,\max\left\{x_2(\tau)\,|\,\tau\in\left[\lambda_1\theta;\lambda_2\theta\right]\right\}\right)d\theta\right)\right]ds+$$

$$+\sum_{k=1}^{P}G(t,t_k)\left[I_k(x_1(t_k))-I_k(x_2(t_k))\right].\quad(21)$$

Now, using the conditions of the theorem, similarly to the estimate (15) from (21) we obtain

$$\left|x_1(t)-x_2(t)\right|\le Q^{-1}(t)\left[B_1(t)-B_2(t)\right]+\int_0^T\left|G(t,s)\right|\cdot\left[M_1(s)\cdot\left|x_1(s)-x_2(s)\right|ds+\right.$$

$$+M_2(s)\int_0^T M_3(s,\theta)\times$$

$$\times\left|\max\left\{x_1(\tau)\,|\,\tau\in\left[\lambda_1\theta;\lambda_2\theta\right]\right\}-\max\left\{x_2(\tau)\,|\,\tau\in\left[\lambda_1\theta;\lambda_2\theta\right]\right\}\right|d\theta\Big]ds+$$

$$+\sum_{i=1}^{p}\left|G(t,t_i)\right|\cdot m_i\cdot\left|x_1(t_i)-x_2(t_i)\right|$$

or passing the norm, we obtain from last that

$$\left\|x_1(t)-x_2(t)\right\|_{PC}\le\left\|Q^{-1}(t)\right\|\left\|B_1(t)-B_2(t)\right\|+\rho\cdot\left\|x_1(t)-x_2(t)\right\|_{PC}.$$

According to the last conditions of the theorem $\rho<1$. So, from the last inequality follows that

$$\|x_1(t)-x_2(t)\|_{PC}\le(1-\rho)^{-1}\left\|Q^{-1}(t)\right\|\cdot\|B_1(t)-B_2(t)\|.$$

If we suppose that $\max\limits_{0\le t\le T}\|B_1(t)-B_2(t)\|<\delta$, then from last estimate we obtain small difference $\|x_1(t)-x_2(t)\|_{PC}<\varepsilon$, where $\varepsilon=(1-\rho)^{-1}\max\limits_{0\le t\le T}\left\|Q^{-1}(t)\right\|\cdot\delta.$ The theorem is proved.

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