PURSUIT AND EVASION DIFFERENTIAL GAMES ON THE 1-SKELETON OF TESSERACT

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Abstract. *In the paper, we study pursuit and evasion differential games within a four-dimensional cube, where all the players move along the edges. The problem is to find the optimal number of pursuers in the game, to construct strategies for the pursuers in pursuit game, and evasion strategy in evasion game.*
Keywords: *tesseract, differential game, pursuer, evader, strategy*

1 Introduction

The notion of differential game was introduced by Isaacs in1965, and then, Pontryagin and Krasovskii gave fundamental contribution to the theory of differential games creating the formalizations to the theory. The theory was further developed by many researchers such as Azamov, Berkovitz, Satimov and others (see, e.g., [1] and references therein).

One of the most significant current discussions in differential games is multi player differential games. In recent years, there has been an increasing interest in differential games of several players (see, e.g., [2] and references therein).

However, if exhaustable resources such as energy, fuel, resources etc. are restricted for the modeling control processes, then control functions are restricted by integral constraints. The method of resolving functions for games with integral constraints was used to obtain a sufficient condition in solving a pursuit differential game in several works (see, e.g., [3] and references therein).

Some games with either geometric or integral constraint, restrict the movement of players to some specific state constraints. For examples, differential games in a convex subset of were studied in [4]. Furthermore, differential games within a geometrical structure in the form of abstract graphs as its state constraint, are of increasing interest. These types of games have minimax forms of which, each being a model for the search problem of a moving object, as mentioned in [5].

In this type of game, players move from one vertex to its adjacent vertex by jumping constitute one type. Another type of game on abstract graphs is where players move along the edges of a given graph embedded in a Euclidean space, as studied in [6-10].

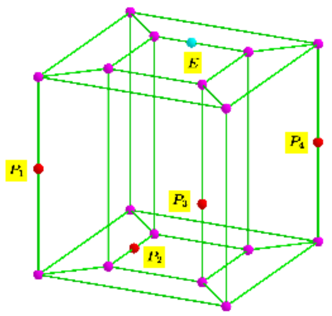

Figure 1: The graph of four dimensional cube K.

One of the most recent work involved a study in a differential game of many pursuers and one evader within 1-skeleton graph of an orthoplex of dimension $d+1$, as discussed in [10].

The current paper intends to study both pursuit and evasion differential games within a four-dimensional cube. All the players move along the edges of the cube and the search for the optimal number of pursuers to ensure pursuit can be completed, are also done.

2 Statement of problem

We consider a differential game of n pursuers x_1, $x_2,...,$ x_n, $n \geq 2$, and one evader y whose dynamics are given by the following equations

$$\begin{aligned} &\dot{x}_i = u_i, \quad x_i(0) = x_{i0}, \; i = 1,...,n, \\ &\dot{y} = v, \quad\;\; y(0) = y_0, \end{aligned} \tag{1}$$

where $x_{i0}, y_0 \in K$, $x_{i0} \neq y_0$, $i = 1,...,n$; u_i is the control parameter of i-th pursuer, and v is the control parameter of the evader. All the players move along the edges of four-dimensional cube K. The maximal speeds of the pursuers x_1, $x_2,...,$ x_n are ρ_1, $\rho_2,...,$ ρ_n, respectively, and that of evader y is 1, i.e., $|u_i| \leq \rho_i$, $i = 1,...,n$, $|v| \leq 1$. It is assumed that $1/3 \leq \rho_i < 1$.

We let $B(r)$ denote the ball of radius r and centered at the origin of the Euclidean space R^{d+1}.

Definition 1. A measurable function $u_i(\cdot)$, $u_i : [0, \infty) \to B(\rho_i)$ is called admissible control of the i-th pursuer, $i \in \{1,...,n\}$, if for the solution $x_i(\cdot)$ of the equation

$$\dot{x}_i = u_i, \, x_i(0) = x_{i0},$$

we have $x_i(t) \in K$, $t \geq 0$.

Definition 2. A measurable function $v(\cdot)$, $v : [0, \infty) \to B(\sigma)$ is called admissible control of the evader, if for the solution $y(\cdot)$ of the equation

$$\dot{y} = v, \, y(0) = y_0,$$

we have $y(t) \in K$, $t \geq 0$.

We consider pursuit and evasion differential games. In the pursuit differential game pursuers apply some strategies and evader uses an arbitrary admissible control. Let us define strategies of pursuers.

Definition 3. The functions $(t, x_1,..., x_n, y, v) \to U_i(t, x_1,..., x_n, y, v)$, $i = 1, 2,..., n$, are called strategies of pursuers x_i, $i = 1, 2,..., n$, if the initial value problem (1) has a unique solution $x_1(t),..., x_n(t)$, $y(t) \in K$, $t \geq 0$, for $u_i = U_i(t, x_1,..., x_n, y, v)$, $i = 1, 2,..., n$, and for any admissible control $v = v(t)$ of the evader.

Definition 4. If, for some number $T > 0$, there exist strategies of pursuers such that $x_i(\tau) = y(\tau)$ at some τ, $0 < \tau \leq T$ and $i \in \{1,...,n\}$, then pursuit is said to be completed. The pursuers are interested in completing the pursuit as earlier as possible.

Definition 5. A function $(t, x_1,..., x_n, y) \to V(t, x_1,..., x_n, y)$ is called a strategy of the evader y if the initial value problem (1) has a unique solution $x_1(t),..., x_n(t)$, $y(t) \in K$, $t \geq 0$, for $v = V(t, x_1,..., x_n, y)$ and for any admissible controls of pursuers $u_i = u_i(t)$, $i = 1, 2,..., n$.

Definition 6. If, for some initial states of players $x_{10},...,x_{n0},y_0 \in K$, there exists a strategy of evader such that $x_i(t) \neq y(t)$ for all $t \geq 0$, and $i = 1,2,...,n$, then we say that evasion is possible in the game in K.

The evader is interested in maintaining the inequality $x_i(t) \neq y(t)$ as long as possible. Since for some initial states the evader may be trapped by pursuers and pursuit can be completed by pursuers easily, therefore this definition contains the phrase "for some initial states of players $x_{10},...,x_{n0},y_0 \in K$".

The number $N = N(K)$ is called the optimal number of pursuers for the game on the cube K if, for any initial states of players, pursuit can be completed in the game with N pursuers and evasion is possible in the game with $N-1$ pursuers.

The problem is to find the optimal number of pursuers N in the game, to construct strategies for the pursuers in pursuit game, and evasion strategy in evasion

3 Main Result

Without any loss of generality we assume that the lengths of edges of the cube K is equal to 1.

We obtained the following statements.

Theorem 1 (Pursuit differential game). Four pursuers x_1,x_2,x_3,x_4 can complete the pursuit in the differential game on 1-skeleton of the four dimensional cube K.

Theorem 2 (Evasion differential game). Evasion from three pursuers x_1,x_2,x_3 is possible in the differential game on 1-skeleton of the four dimensional cube K.

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